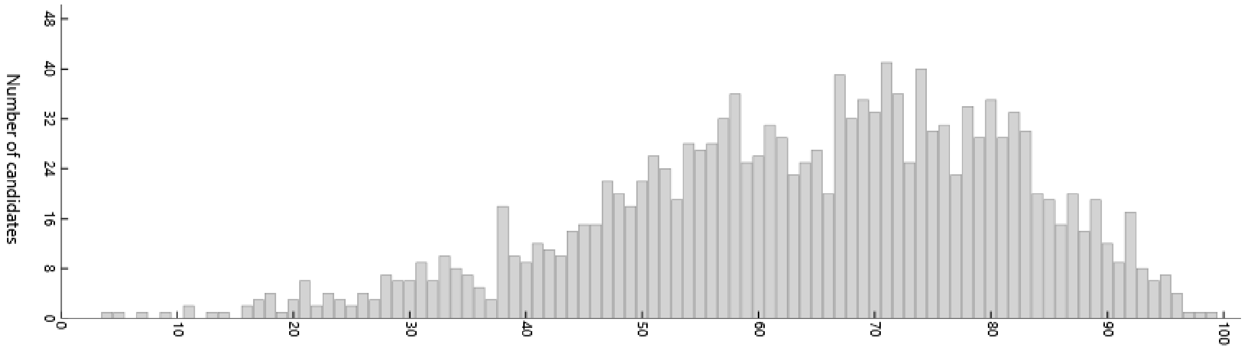
2017 ATAR course examination report: Mathematics Specialist

Year	Number who sat	Number of absentees
2017	1463	12
2016	1427	17

Examination score distribution – Written



Summary

Attempted by 1463 candidates Mean 63.95% Max 98.68% Min 4.30%
The examination consisted of two sections: a Calculator-free section and a Calculator-assumed section.

Section means were:
Section One: Calculator-free Mean 62.58%
Attempted by 1463 candidates Mean 21.90(/35) Max 35.00 Min 1.32
Section Two: Calculator-assumed Mean 64.68%
Attempted by 1463 candidates Mean 42.04(/65) Max 65.00 Min 0.67

General comments

The paper appeared to be well received, with all questions being accessible and with sufficient material to challenge the more able candidates. The Calculator-free section was more difficult than that of the 2016 paper. The Calculator-free mean was 62.58% and the Calculator-assumed mean was 64.68%.

There was no significant evidence that candidates had a problem with the length of the paper, as shown by the high attempt rates for questions at the end. This year, there were fewer candidates writing exclusively with a pencil and more using a pen, albeit that many did not write legibly, particularly with respect to forming digits.

Sections of the course that were generally not understood well are listed below.

- In Question 4 (d) of Section One, to determine the domain and range of a composite function required ability to write and solve inequalities. A reluctance to form an inequality in Question 17 (d) underscores this.
- The understanding of the notion of a vector within (or in) a plane as against a position vector specifying a point in a plane. Many candidates demonstrated genuine confusion with this concept as seen in Question 7 (a) (ii) and in Question 15 (a). Combined with this confusion was a lack of attention to vector notation that made a candidate’s task much more difficult, as seen throughout Question 15.

- In Section Two, throughout Question 13 candidates had difficulty in writing sensible statements, particularly with respect to the sampling of the mean and confidence intervals.
- The ability to recognise standard differential equations, specifically for simple harmonic motion and exponential decay.
- The general reluctance to use implicit differentiation to determine rates, as seen in Questions 12 (a) and 18 (c). Implicit differentiation is a key skill in the Mathematics Specialist course.

Advice for candidates

- Write legibly using a ball point pen, particularly taking care to form digits that can be read by markers.
- Show all working and acknowledge where a CAS calculator routine has been used.
- When you are working in the statistics section, write mathematical statements, not language specific to a CAS calculator.
- Acknowledge that a variable is normally distributed and show clearly the parameters (mean and standard deviation) used.
- When questions are worth more than two marks, do not simply write an answer but show valid working or justification to receive full marks.

Advice for teachers

- Provide students with many opportunities to explain ideas, using appropriate mathematics language and using correct mathematical notation.
- Ensure students understand the importance of the legibility of their work, the need to show all working and to write clear mathematics statements rather than language specific to a CAS calculator.
- Focus students' conceptual understanding with vectors. It is a clear advantage if teachers can show students (using appropriate computer software), vector ideas in three dimensions; and also to insist on the correct use of vector notation, to distinguish between many vector ideas.

Comments on specific sections and questions

Section One: Calculator-free (53 Marks)

Candidates performed well in the in the following areas:

- showing a linear divisor is indeed a factor of a polynomial in Question 2 (a)
- sketching the graph of an inverse function in Question 4 (a) and determining the inverse rule in Question 4 (b)
- sketching the graph of a rational function in Question 5.

Question 1 attempted by 1441 candidates Mean 2.88(/4) Max 4 Min 0

Most candidates expressed *w* as a complex number having equal real and imaginary parts, with then a majority able to deduce the argument in the first quadrant. A minority deduced both possible arguments which meant the first question enabled discrimination between candidates. Those candidates who could not work with the conjugate correctly invariably scored poorly.

Question 2 attempted by 1457 candidates Mean 5.00(/6) Max 6 Min 0

Part (a) was a very similar question to one asked in last year's paper and candidates did better in showing that the result of the substitution was in fact zero. In part (b), most candidates realised that the conjugate value was involved to find the other factor. In part (c), many candidates, despite knowing two factors, did not state the corresponding solutions. Some candidates did not factorise correctly. Many who did factorise, then did not solve the equation.

Question 3 attempted by 1451 candidates Mean 4.65(/7) Max 7 Min 0
In part (a), candidates knew what they needed to do (in this technique question), but many complicated the issue by wanting to write the derivative in terms of cosine, when secant made it much simpler. Part (b) was done reasonably well, yet still many candidates wished to integrate incorrectly $\sin^2 u$ as $\frac{\sin^3 u}{3}$.

Question 4 attempted by 1463 candidates Mean 6.59(/9) Max 9 Min 0
The quality of graphing the inverse function was excellent in part (a). For part (b), determining the inverse rule was done quite well, yet many candidates did not state the domain. Part (c) was done well and errors in simplification were not penalised. Finding the domain of the composite function in part (d) was a challenging question for many candidates, with the solving of an inequality using algebraic fractions proving difficult.

Question 5 attempted by 1450 candidates Mean 4.02(/6) Max 6 Min 0
Some of the concepts involved in this question were not fully understood by some candidates. The identification of the horizontal asymptote $y=-4$ was frequently not found or somehow determined to be $y=4$ suggesting that some candidates may not have read the opposite sign in the defining rule. However, many candidates correctly plotted either the turning point at $x=1$ or the vertical intercept to give detail to their graph.

Question 6 attempted by 1441 candidates Mean 2.50(/6) Max 6 Min 0
In part (a), a large number of candidates used vector notation instead of the form indicated in the question. Candidates should be encouraged to write statements using the complex number z. A significant number of candidates mistakenly thought that the radius was 2 units. Part (b) was one of the more challenging questions for the cohort. However, the question allowed many original and well thought-out responses.

Question 7 attempted by 1440 candidates Mean 6.03(/10) Max 10 Min 0
Part (a) (i) was a straightforward question requiring the vector equation of a line. Many candidates interpreted that finding the direction vector was all that was required. Part (a) (ii) was generally done poorly. Candidates need to know the difference between vectors that are in the plane as distinct from points that lie in the plane. This confusion about vectors was similarly seen in Q15 (a). Candidates must also be aware that if the cross product is to be used to find the normal vector, then two non-parallel vectors must be used. Those candidates who obtained a vector equation for the plane usually did not convert to a Cartesian equation as required. Part (b) saw a better performance using the dot product expression.

Question 8 attempted by 1394 candidates Mean 1.84(/5) Max 5 Min 0
In part (a), candidates had to express x as the subject in their defining rule. Many candidates could not correctly determine the limits of integration, often using a,b as the respective lower and upper limits, rather than $0,h$. Part (b) was found to be challenging for all but the very top candidates. Errors occurred mainly from a lack of facility with algebra, not expanding correctly, not anti-differentiating correctly or not simplifying.

Section Two: Calculator-assumed (97 Marks)
Candidates performed well in the in the following areas:

- representing complex numbers as vectors in Question 10
- determining an area between curves in Question 12 (b)
- determining the confidence interval for a population mean based on a sample in Question 13 (a)
- writing a defining rule using an absolute value function in Question 16 (a)
- solving an equation in the complex plane in Question19 (a).

Question 9 attempted by 1440 candidates Mean 3.86(/6) Max 6 Min 0
In part (a), candidates needed to state the sample mean as being normally distributed, using correct mathematical notation, in order to secure full marks. In part (b), candidates did not take into account the relatively small standard deviation for the sample mean. Most curves showed a much wider spread. Many candidates also did not indicate their answer from part (a) on their graph, as instructed.

Question 10 attempted by 1458 candidates Mean 5.36(/7) Max 7 Min 0
The first three parts of the question were done well by candidates, except for some who simply wanted to add/subtract $\frac{\pi}{2}$ without changing the modulus. For part (d), requiring a description of the transformation, it appeared that many candidates may not have heard of the phrase 'clockwise rotation' and were not well-equipped to use appropriate mathematics vocabulary.

Question 11 attempted by 1452 candidates Mean 5.48(/7) Max 7 Min 0
In part (a), the majority of candidates knew that the derivative needed to be evaluated and did so correctly. A number of candidates understood that an undefined gradient meant that a tangent had to be vertical. Quite a number of candidates proceeded in part (c) to find the equation of the tangent $y=-\frac{x}{2}+1$, indicating that they did not understand that the equation of the line of force (the curve) was required. The absolute value of the natural logarithm function was missed by many, which then caused them to write an integration constant which was not a real number $\ln(-2)$. Some candidates acknowledged that since $x<1$, then they could write $y=-\frac{1}{2}\ln(2-2x)+c$ without having to use the absolute value.

Question 12 attempted by 1460 candidates Mean 6.63(/8) Max 8 Min 0
Part (a) was done well. Some candidates who chose not to differentiate implicitly, then made algebraic errors in expanding $y=\left(3-\sqrt{x}\right)^2$. Part (b) was answered quite well, permitting many avenues to a correct solution. Candidates generally used their CAS calculator to evaluate integrals, yet many were content to do so without. Some candidates considered the region as a sum of horizontal rectangles, which was the easier approach using a single definite integral.

Question 13 attempted by 1452 candidates Mean 8.22(/13) Max 13 Min 0
In part (a), many candidates wrote the solution without providing supporting evidence. The common error was using the standard deviation as 400, with many using 4000 for some reason. For part (b) (i), failure to answer the question with true or false plagued many, along with a general lack of understanding. Candidates seemed ill-prepared for questions requiring explanation and were more prepared to quote rote statements such as '95% of the time the sample mean will fall into the confidence interval' and so did not directly answer the

question. It should be realised that it is not true that 95% of sample means will fall into the calculated confidence interval from part (a). A better answer was to say that it is not true, and that by random sampling, we are not guaranteed that the sample mean will fall into a specified confidence interval. A similar comment applies for part (b) (ii). A single observation will have a much larger variance, and so is very unlikely to fall within the confidence interval for a mean. Exactly what is normal (or becomes normal) was clearly misunderstood by most candidates. Language needs to be precise here; simply saying ‘it is normal’ or ‘the sample is normal’ or ‘the data is normal’ does not indicate an understanding of the issue. Indeed, regardless of sample size the sample data is not normal if the population distribution is normal. So it needs to be clearly stated that for large sample sizes $(n \geq 30)$ the sample mean is approximately normal. In part (d), performance was better. However, some candidates could not find the confidence level from their critical z score.

Question 14 attempted by 1441 candidates Mean 4.57(/7) Max 7 Min 0
Candidates generally performed well in part (a). Errors were in not substituting $t = 120$ seconds or assuming that the magnitude of the vector was required. Part (b) was done reasonably well, yet the order of subtraction was an issue for many candidates. Despite many answering this well, a number of candidates in part (c) assumed that the closest approach was being required. Many did not state explicitly whether interference would be caused or not.

Question 15 attempted by 1446 candidates Mean 7.54(/13) Max 13 Min 0
Few candidates drew the vector required. Instead, they plotted the point (0,2) showing a dot. This indicates a misconception as to what constitutes a vector, which is similar to the confusion seen with Q7(a) (ii). Candidates had difficulty in choosing the correct expression to answer the question. Considerable confusion was exhibited between the meaning of the following in part (b): $\int_a^b \underset{\sim}{v}(t)\,dt,\ \int_a^b \left|\underset{\sim}{v}(t)\right|dt,\ \left|\int_a^b \underset{\sim}{v}(t)\,dt\right|$. To receive full marks in this question, correct mathematics notation was required.

For part (c) to receive full marks, candidates had to show that they used the initial position vector to determine the vector constant to determine the expression for $\underset{\sim}{r}(t)$. Finding the acceleration vector in part (d) was problematic for many candidates who did not realise that the value for t had to be found to match the given position on the curve. Many candidates were only able to differentiate the expression. Some candidates stated their answer as a scalar, which was clearly not the question. Part (e) was very challenging for many candidates. Commonly they determined the time but did not evaluate the expression for distance correctly.

Question 16 attempted by 1443 candidates Mean 5.20(/8) Max 8 Min 0
Part (a) was done well by most candidates. A variety of approaches was used. In part (b), many candidates ignored the clear instruction to use the substitution to change the variable, instead going straight to use their CAS calculator, as evidenced by those who had answers with $|b|$ in it. A small number identified correctly that the integral was simply an area under the graph.

Question 17 attempted by 1438 candidates Mean 7.35(/11) Max 11 Min 0
The first part of the question was relatively straightforward, requiring candidates to recognise the differential equation indicating simple harmonic motion. In part (b), many candidates opted to use an integral of the absolute value of displacement, indicating confusion as to how to measure the speed. In part (c), too few candidates could recognise the differential equation indicating an exponential function. In part (d), most candidates made no allowance for the fact that the amplitude had to be less than 0.01. Few candidates rounded the critical time value up to one decimal place.

Question 18 attempted by 1394 candidates Mean 5.28(/10) Max 10 Min 0
Parts (a) and (b) were answered well. Many candidates did not differentiate implicitly as instructed in part (c). Candidates had to correctly evaluate s and θ when $t=4$ and then report their value correct to two decimal places. Part (d) was a more challenging question. Those candidates who approached it by defining $\frac{ds}{dt}$ as their function and then graphing this to determine a maximum were mostly successful. Some candidates explained that the maximum rate would occur when the tangent position was achieved, enabling an exact value for $\cos\theta$ to be achieved using right triangle trigonometry.

Question 19 attempted by 1434 candidates Mean 4.13(/7) Max 7 Min 0
Solving the complex equation using polar form was done well, yet many candidates opted unnecessarily to give all six solutions. Part (b) was a challenging question. There were some very elegant solutions which attracted full marks. Some candidates used a trial and error method and could score two marks but did not justify why only four possible values existed.